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September 1963

On the Effect of Wrinkles on the

Reflections by a Sphere

by

F. J. Tischer

SUMMARY

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Scattering of microwaves by a wrinkled spherical surface is considered. First, a rigorous formulation of the problem is considered. Second, approximate solutions based on geometrical and physical optics are presented.

AUTHOR

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I. Introduction

In using a sphere as a passive reflector in space communications, the knowledge of the effect of a wrinkled surface on the reflection properties is evidently important. The following considerations may serve as introductory dealing with this problem.

A large reflecting sphere (radius $a \approx 30$m, operational wavelength of the radio waves $\lambda_o \approx 6$cm) is being assumed. First, the problem is formulated rigorously. Since the solution in this formulation is rather complicated, in the second part, an approximate solution by a mixed geometrical- and physical-optics approach is considered.

II. Rigorous Formulation:

The problem of the effect of wrinkles can be formulated rigorously by considering scattering by a sphere with a perfectly conducting surface, the radius of which deviates by ΔR from an average radius a, where ΔR is a function of θ and ϕ (see Fig. 1) and is assumed to be known,

$$a' = a + \Delta R(\theta, \phi). \tag{1}$$

The effect of the wrinkles is found by determining the electric or magnetic field intensity scattered by the surface according to Eq. (1) for incident plane waves.

We write for the incident waves traveling in the z-direction toward the sphere

$$E_x^i = E_o \exp j(wt - \beta_o z) = E_o \exp j(wt - \beta_o r \cos\theta), \tag{2a}$$

$$H_y^i = E_x^i / Z_o . \tag{2b}$$

Fig. 2 shows the geometry of the initial conditions. We express the plane waves by a superposition of spherical TE and TM waves described by the radial components of a magnetic (A_r) and an electric vector potential (F_r) respectively,

$$A_r^i = \frac{E_o \beta_o r}{w\mu} \cos\phi \sum_{n=1}^{\infty} \frac{j^{-n}(2n+1)}{n(n+1)} j_n(\beta_o r) P_n^1 (\cos\theta), \tag{3a}$$

$$F_r^i = E_o r \sin\phi \sum_{n=1}^{\infty} \frac{j^{-n}(2n+1)}{n(n+1)} jn(\beta_o r) P_n^1 (\cos\theta). \tag{3b}$$

Without wrinkles, the boundary conditions ($H_r = 0$; E_θ, $E_\phi = 0$ for $r = a$) postulate that the total field becomes

$$A_r^{tot} = A_r^i + A_r^s = \frac{E_o \beta_o r}{w\mu} \cos\phi \sum_{n=1}^{\infty} \frac{j^{-n}(2n+1)}{n(n+1)} \left[j_n(\beta_o r) + k_A h_n^{(2)}(\beta_o r) \right] P_n^1 , \tag{4a}$$

$$F_r^{tot} = F_r^i + F_r^s = E_o r \sin\phi \sum_{n=1}^{\infty} \frac{j^{-n}(2n+1)}{n(n+1)} \left[j_n(\beta_o r) + k_F h_n^{(2)}(\beta_o r) \right] P_n^1 , \tag{4b}$$

where

$$k_A = -\frac{j_n'(\beta_o a)}{h_n^{(2)\prime}(\beta_o a)} \quad ; \quad k_F = -\frac{j_n(\beta_o a)}{h_n^{(2)}(\beta_o a)} .$$

The quantities j_n and $h_n^{(2)}$ are spherical Bessel and Hankel functions respectively, and P_n^1 are Legendre functions.

The scattered field solely is then given by

$$\begin{matrix} A_r^s \\ \\ F_r^s \end{matrix} = \begin{matrix} \dfrac{E_o \beta_o r}{w\mu} \cos\phi \\ \\ E_o r \sin\phi \end{matrix} \sum_{n=1}^{\infty} \begin{matrix} -\dfrac{j_n'(\beta_o a)}{h_n^{(2)'}(\beta_o a)} \\ \\ -\dfrac{j_n(\beta_o a)}{h_n^{(2)}(\beta_o a)} \end{matrix} h_n^{(2)}(\beta_o r)\, P_n^1 \; . \tag{5}$$

With wrinkles, we have to modify the boundary conditions such that:

$$\text{for} \quad r = a + \Delta R(\theta, \phi) \qquad \begin{aligned} \bar{n} \cdot \bar{H} &= 0, \\ \bar{n} \times \bar{E} &= 0. \end{aligned}$$

These conditions lead to complicated equations. The problem can be simplified under the assumption that the slope of the function $\Delta R(\theta, \phi)$ is small, where

$$\bar{n}\,(\theta, \phi) \cdot \bar{a}\,(\theta, \phi)/a \; - \; 1 \ll 1.$$

With this assumption, the scattered field can be described by

$$A_r^{s'} = A_r^s \exp.\,[\,-j\,4\,\pi\,\Delta R(\theta, \phi\,)/\lambda_o] \tag{6a}$$

and

$$F_r^{s'} = F_r^s \exp.\,[\,-j\,4\,\pi\,\Delta R(\theta, \phi\,)/\lambda_o\,]. \tag{6b}$$

At an arbitrary distance, the field resulting from scattering by the sphere with wrinkled surface becomes

$$\begin{matrix} A_r(r) \\ F_r(r) \end{matrix} = \sum_{n=0}^{\infty} \begin{matrix} a_{on} \\ c_{on} \end{matrix} P_n(\cos\theta) + \sum_{n=1}^{\infty} \sum_{m=1}^{n} \left[\begin{matrix} a_{nm} \\ c_{nm} \end{matrix} \cos m\phi \right.$$

$$\left. + \begin{matrix} b_{nm} \\ d_{nm} \end{matrix} \sin m\,\phi \right] h_n^{(2)}(\beta_o r)\; P_n^{\,m}(\cos\theta). \tag{7}$$

The constants a_{nm}, b_{nm}, c_{nm}, d_{nm} are found from the following equations:

$$\begin{matrix} a_{on} \\ c_{on} \end{matrix} = \frac{2n+1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \begin{matrix} A_r^{s} \\ F_r^{s} \end{matrix} \exp[-j4\pi\,\Delta R(\theta,\phi)/\lambda_o]\; P_n(\cos\theta)\,\sin\theta\, d\theta\, d\phi; \tag{8a}$$

$$\begin{matrix} a_{nm} \\ c_{nm} \end{matrix} = \frac{2n+1}{2\pi}\, \frac{(n-m)!}{(n+m)!} \int_0^{2\pi} \int_0^{\pi} \begin{matrix} A_r^{s'} \\ F_r^{s'} \end{matrix} P_n^{\,m}(\cos\theta)\, \cos\phi\, \sin\theta\, d\theta\, d\phi; \tag{8b}$$

$$\begin{matrix} b_{nm} \\ d_{nm} \end{matrix} = \frac{2n+1}{2\pi}\, \frac{(n-m)!}{(n+m)!} \int_0^{2\pi} \int_0^{\pi} \begin{matrix} A_r^{s'} \\ F_r^{s'} \end{matrix} P_n^{\,m}(\cos\theta)\, \sin\phi\, \sin\theta\, d\theta\, d\phi. \tag{8c}$$

The vector potentials yield then the field intensities by using conventional relations [1].

The equations are rather complex and their evaluation becomes difficult. Approximations considering one-dimensional wrinkles, for example wrinkles where ΔR is a function of ϕ only, is somewhat simpler. Further simplification is possible by computing the cylindrical case considering wrinkles in the circumferential direction only.

III. Optical Approximation

A drastic simplification using a mixed geometrical and physical optics approach allows rough estimates of the effect of wrinkles. The simplified model is based on the following assumptions:

1. Large sphere ($a \gg \lambda_o$),

2. Phase shift relations for varying wrinkle wavelengths on the sphere are equal to those in the far-field,

3. Gaussian probability density distribution of the deviations $\Delta R(\theta, \phi)$ from the average radius a.

We substitute the phase shift of the reflected wave elements $\psi(\theta, \phi)$ which corresponds to the deviations $\Delta R(\theta, \phi)$ according to $\psi = 4\pi \; \Delta R/\lambda_o$. A probability density distribution for ψ is introduced, where

$$\int_{-\infty}^{+\infty} p(\psi)\, d\psi = 1.$$

For Gaussian distribution exp. $(-k\psi^2)$, the density distribution has the form

$$p(\psi) = \sqrt{\frac{k}{\pi}} \; \exp \; [-k(4\pi \, \Delta R/\lambda_o)^2], \tag{9}$$

where k is a constant related to the RMS value of ΔR by

$$k = 1/2\,(4\pi\ \Delta R^{RMS}/\lambda_o)^2.$$

The effect of the wrinkles on the amplitude of the reflected field strengths is then, as a first order approximation, given by

$$\int_{-\infty}^{+\infty} \cos\psi \sqrt{\frac{k}{\pi}}\ \exp(-k\ \psi^2)\, d\psi \quad ,$$

which yields

$$E/E_o = \exp.\ (-\ 1/4\ k). \tag{10}$$

Applying a more sophisticated model, the deviations of the phase shift resulting from $\Delta R\ (\theta,\ \phi)$ and varying also with increasing radius within the surface area on the sphere contributing to the reflected field strengths have to be taken into account. In this case, the radial increase of the surface area, which contributes to the reflected field, caused by the wrinkles must be considered also. This increase of the radius is given, as an approximation, by

$$R_o \approx R_{oo} \sqrt{1 + 8\pi\ \Delta R^{RMS}/\lambda_o},$$

where

$$R_{oo} \approx (\lambda_o a/4\pi)^{1/2}$$

is the equivalent radius of the area of the smooth sphere which mainly contributes to the reflected power in direction of the incident waves.

Evaluation of the corresponding integral,

$$E/E_o = 2\sqrt{\frac{k}{\pi}}\ \frac{a^2}{R_o^2} \int_0^{R_o} \int_0^{\psi} \cos\left[\psi - m\left(\frac{R}{a}\right)^2\right] \exp(-k\psi^2)\ \frac{R}{a}\, d\left(\frac{R}{a}\right) d\psi\ , \qquad (11)$$

yields for the reduction of the scattered field strengths as an approximation

$$E/E_o = \exp\,(-1/4k)\ \frac{\sin m\,(R_o/a)^2}{m(R_o/a)^2}\ ; \quad m = 2\pi\, a/\lambda_o\ . \qquad (12)$$

The solutions according to Eqs. (10) and (12) are plotted in Fig. 3, where B represents the better approximation.

It should be noted that the reduction of the field intensities in one particular direction caused by the wrinkles goes hand in hand with increases in other directions, while the total scattered power remains the same. The radiation pattern assumes an irregular form for the wrinkled surface.

The derived results are rough approximations which take into account the specular reflections and which may serve as guide lines for the design of spherical reflectors. More accurate computations using other probability density distributions and attempts to solve the more rigorous formulations of part may lead to more accurate results.

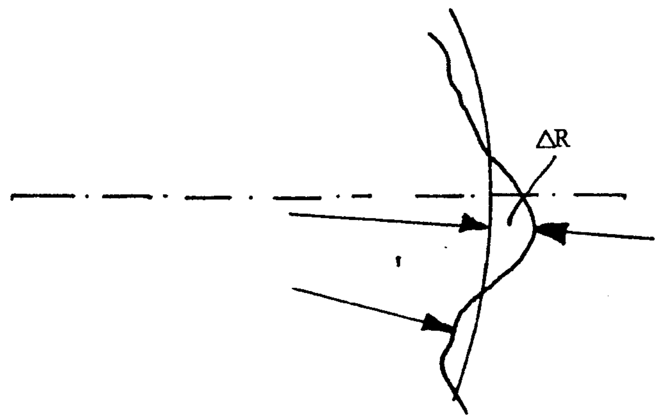

Fig. 1 Wrinkled spherical surface

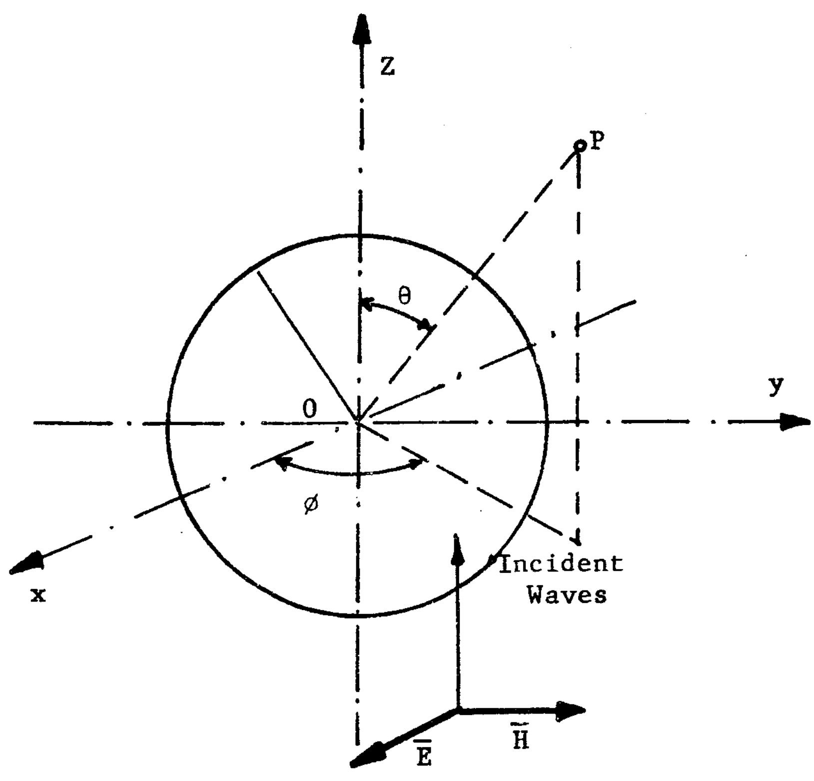

Fig. 2 Geometry of reflections by a sphere

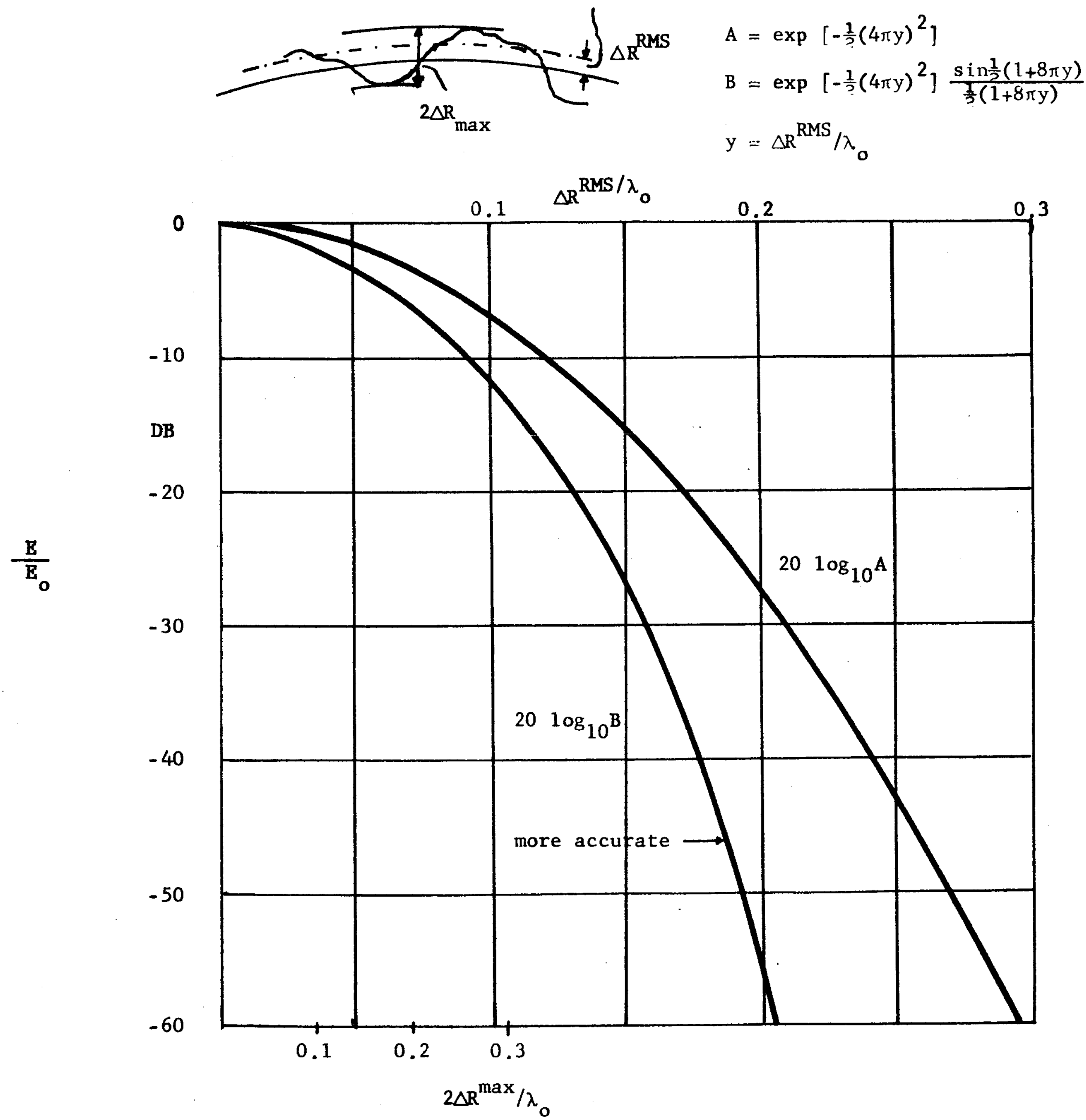

Fig. 3 - Maximum fieldstrength reduction by wrinkles